Random variable: x

Distribution:

$E(x)$ mean

$Var(x)$ variance

$\sigma_x^2 \equiv Var(x)$

$p(x)$ probability

Sample:

$\bar{x}$ sample mean

sample variance

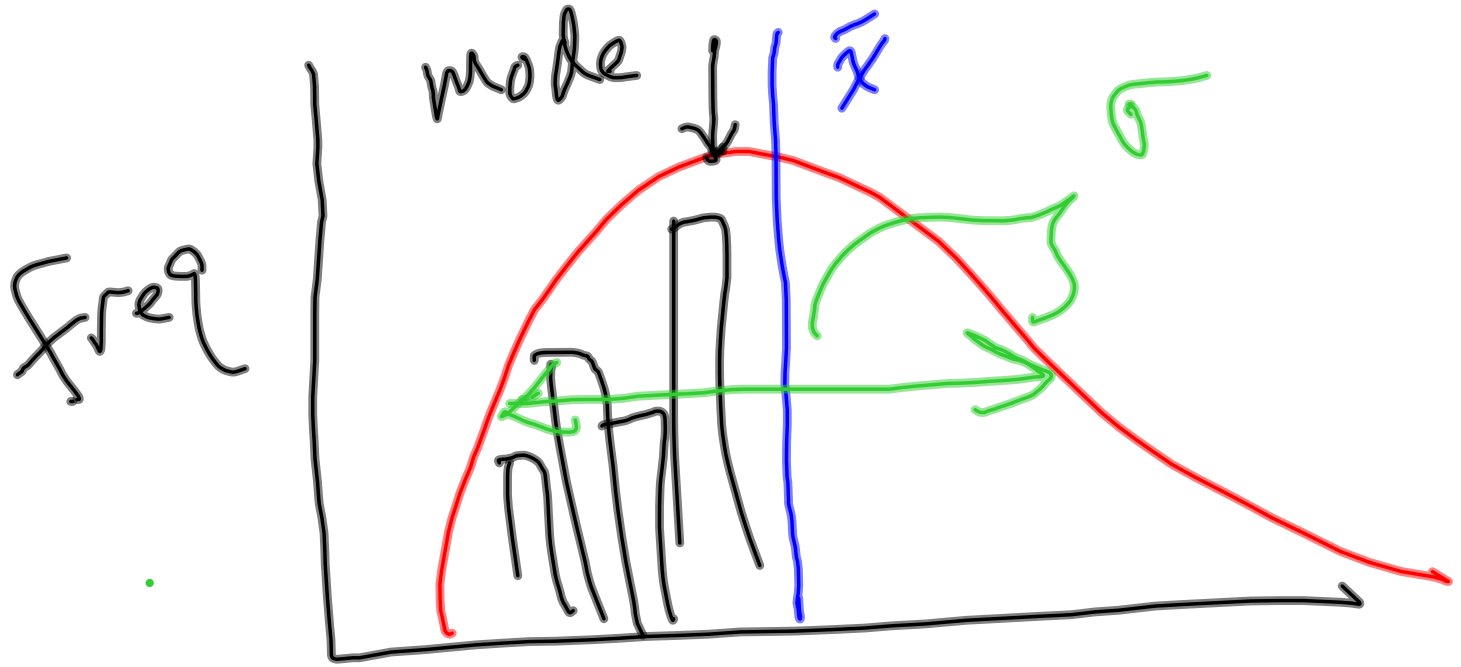
sample frequency

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\sigma_x^2 = \frac{1}{n}\sum_{i=1}^{n} (x_i - \bar{x})^2$$

standard deviation

$$\sigma_x = \sqrt{\sigma_x^2}$$



For example: In a "normal" distribution 95% of the values fall within $\pm 2\sigma$ of $\bar{x}$.

Computation:

$$\sigma_x^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left(x_i^2 - 2x_i\bar{x} + \bar{x}^2\right)$$

$$= \frac{1}{n}\left[\sum_{i=1}^{n}x_i^2 - \sum_{i=1}^{n}2x_i\bar{x} + \sum_{i=1}^{n}\bar{x}^2\right]$$

$$= \frac{1}{n}\sum_{i=1}^{n}x_i^2 - \frac{2\bar{x}}{n}\sum_{i=1}^{n}x_i + \bar{x}^2$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \frac{2\bar{x}}{n}\sum_{i=1}^{n} x_i + \bar{x}^2$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - 2\bar{x}^2 + \bar{x}^2$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \bar{x}^2$$

Correlation

2 random variables x, y

$$Corr_{x,y} = \frac{Covariance(x,y)}{\sigma_x \sigma_y}$$

$$Cov(x,y) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i y_i - \bar{x}\,\bar{y}$$

Serial Correlation

lag 1

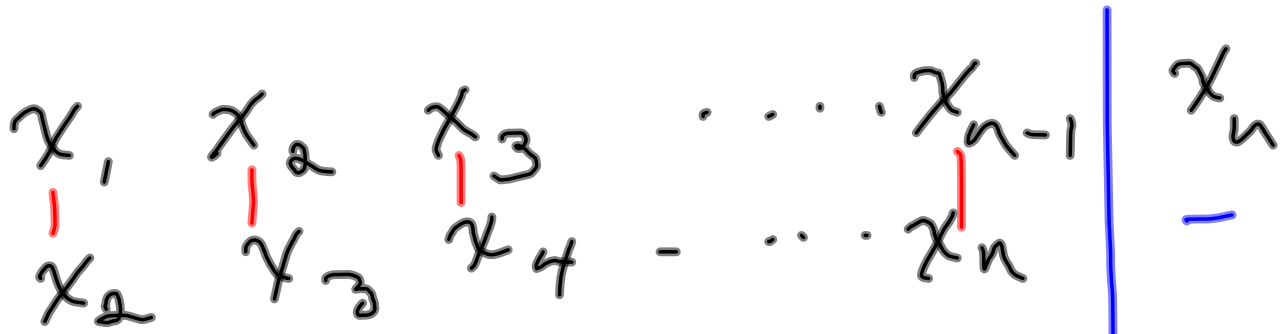
Sample x (order)

$x_1 \; x_2 \; \cdots \; x_n$

Create $y_i = x_{i-1}$

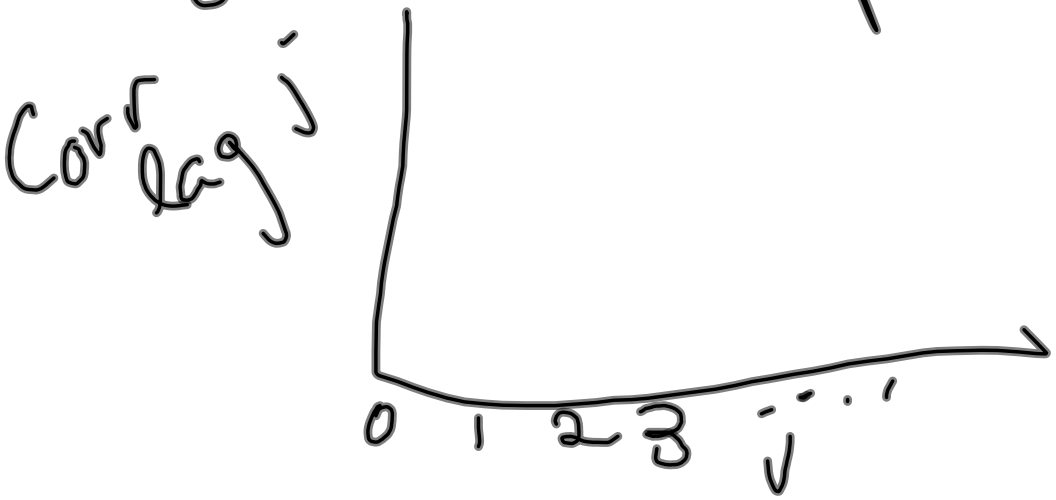
x_1	x_2	x_3	$\cdots$	x_{n-1}	x_n
y_2	y_3	y_4		y_n	

$n-1$ pairs



$(n-1)$ pairs

Lag 2 $(n-2)$ pairs



Distributions

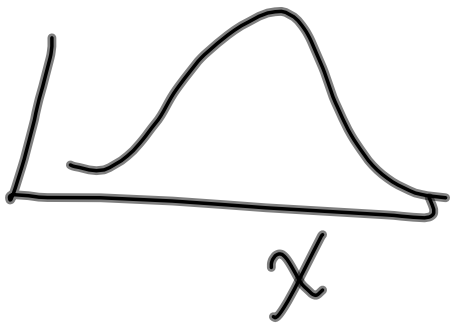
* "shape" distribution function
* integrate function? area under curve
* origin distribution
* simulating distribution

Example

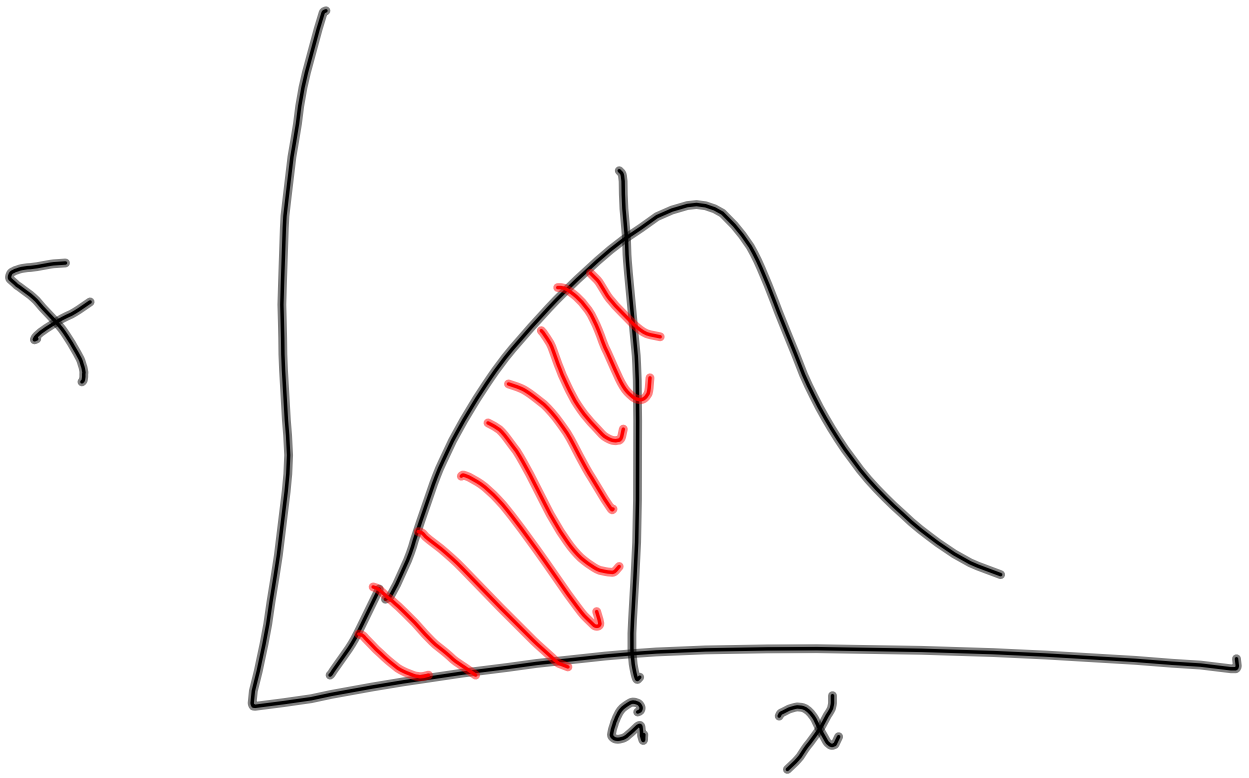
Normal distribution with mean μ and standard dev σ

$$\rho(x) = \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}}$$

Graph ρ



integral = "area under the curve"



numerical integration

examples like

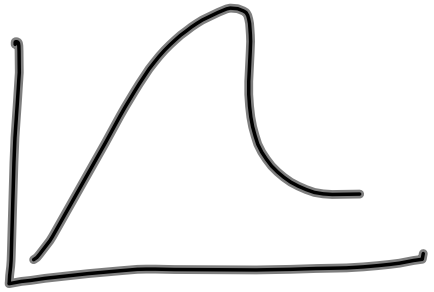
Simpson's rule

Trapezoidal rule

(geometrical)

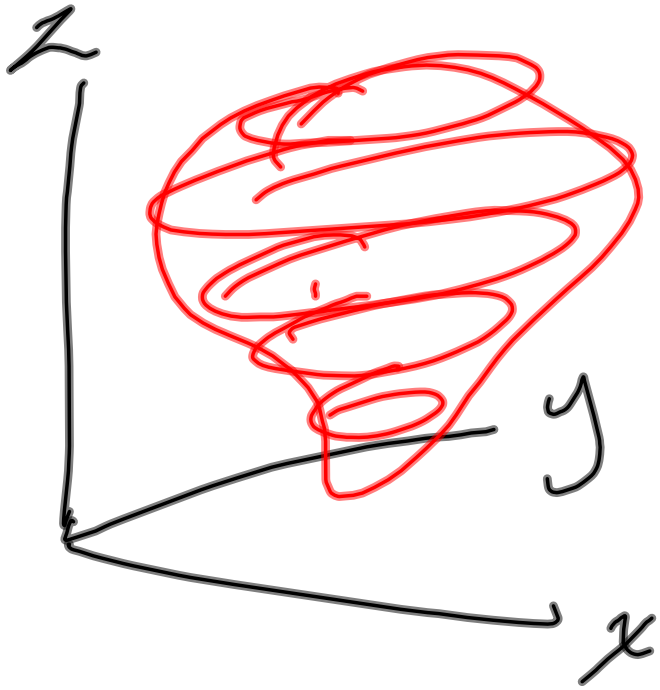
"dimensionality"

$\rho(x, y, z)$

1: ρ 

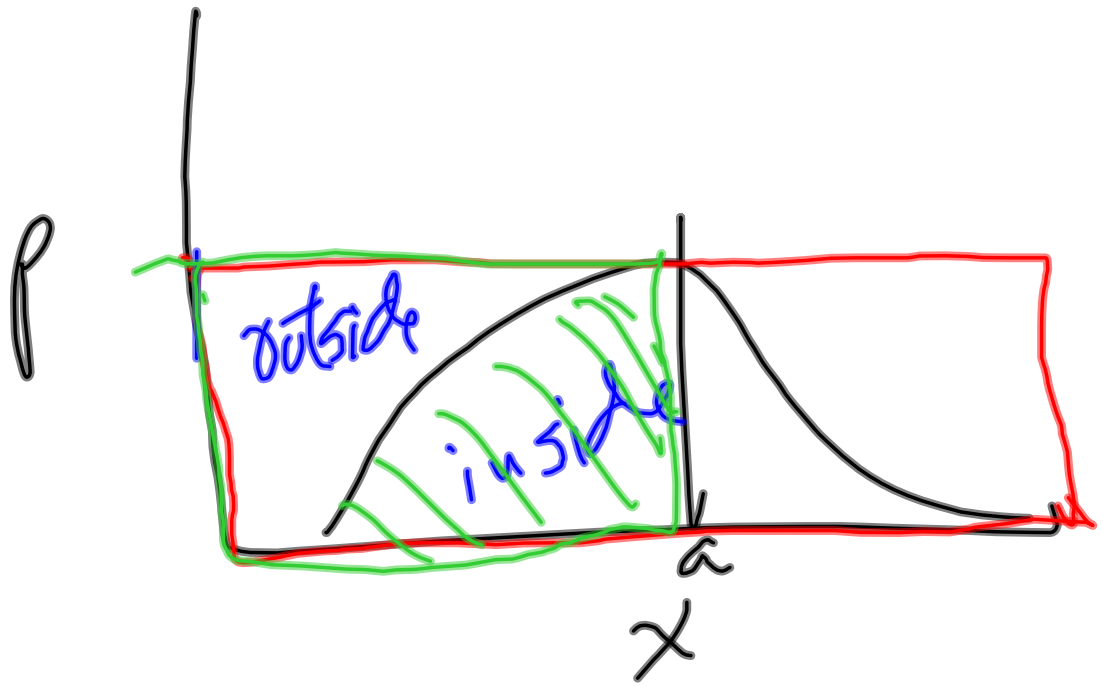
2: ρ 

3:



non-grid numerical integration

"Monte Carlo"



Random number generators

"middle square"

# of digits (even)

Choose 12

will get 6-digit random numbers with uniform distribution

pick arbitrary 6 digit start ("seed")

1. square the 6 digit value
2. represent that as 12 digit number (supply leading zeros as needed)
3. use the middle 6 digits as the next number