Exponential (Geometric) "logarithmic" growth

$$N_t = \lambda N_{t-1}$$

$$N_t = N_0 \, ? \, \lambda^t = N_0 e^{rt}$$

$$N_1 = \lambda N_0$$

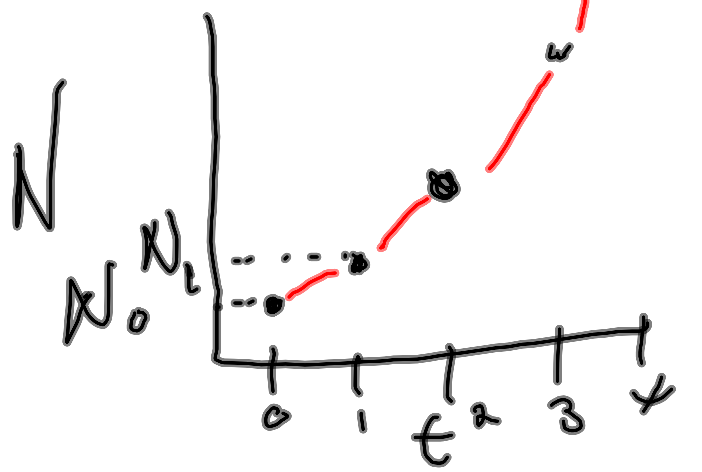
$$N_2 = \lambda N_1 = \lambda \lambda N_0 = \lambda^2 N_0$$

$$N_3 = \lambda N_2 = \lambda \lambda^2 N_0 = \lambda^3 N_0$$

Consider $r = \ln \lambda$

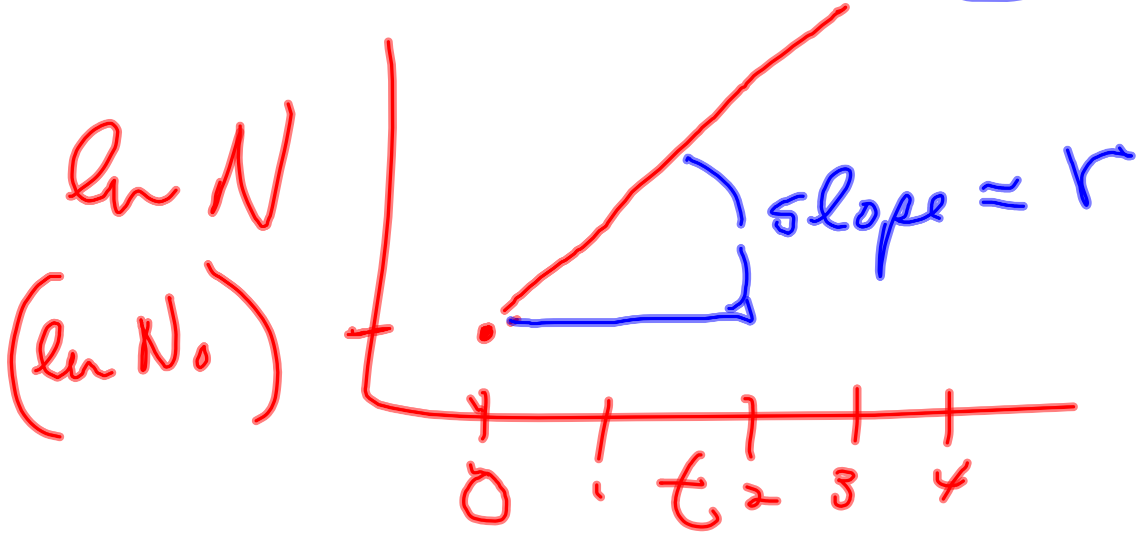
$\lambda = e^r$

Graph N_t against t



$$N_t = N_0 e^{rt}$$

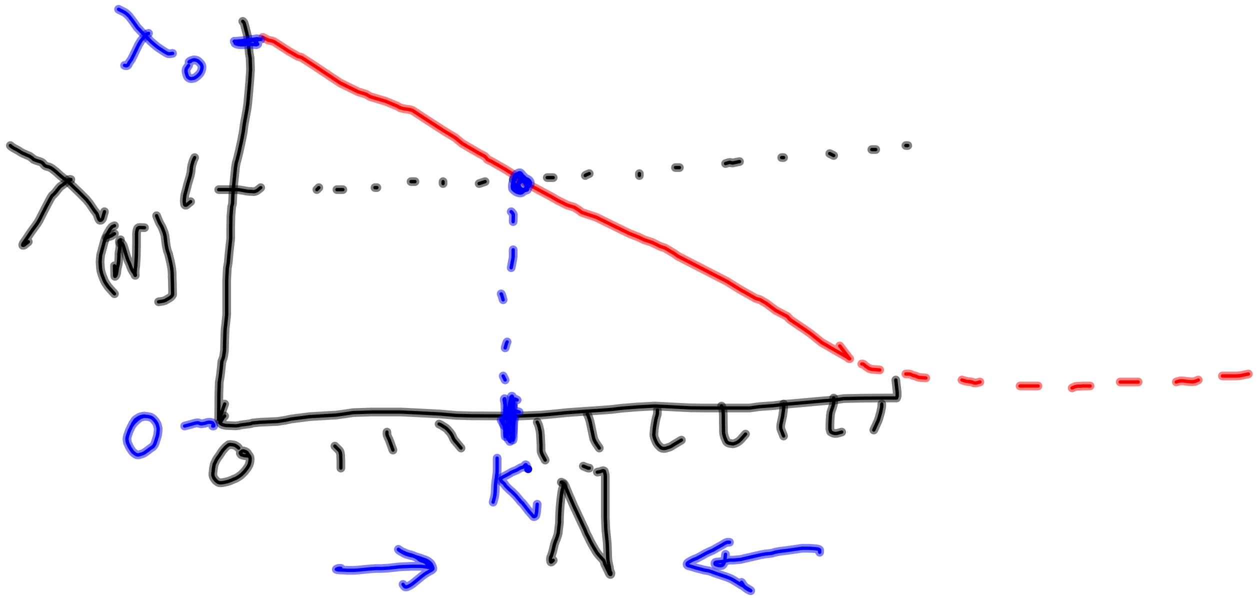
$$\ln N_t = \ln N_0 + r\underline{t}$$



Reference Values

	λ	r
Zero growth	1	0
Positive growth	>1	>0
Decline	<1	<0

Density Dependence



$$r = b - \mu$$

mean per capita birth rate

mean per capita death rate

$$\underline{n} = \begin{pmatrix} n_1 \\ n_2 \\ \vdots \\ n_\omega \end{pmatrix} \qquad \underline{m} = \begin{pmatrix} m_1 \\ m_2 \\ \vdots \\ m_3 \end{pmatrix} \qquad \underline{p} = \begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_{\omega-1} \\ 0 \end{pmatrix}$$

$\underline{n}$ population size at beginning of time step

P_j number of individuals surviving to be censused in age class $j+1$ at the beginning of the next time step, per individual in age class j at the beginning of the present time step

$n_{i,t} P_i = n_{i+1,t+1}$

age class width = time step

m_j number of new individuals surviving to be censused as members of age class 1 at the begining of the next time step per potential parent (♀) alive in age class j at the begining of the present time step

$$n_{1,t+1} = \sum_{j=1}^{\omega} m_j n_{j,t}$$

$$\underline{n}_{t+1} \longleftarrow \underline{n}_t$$

For $i = 2, \omega$

$$n_{i,t+1} = n_{i-1,t}\, p_{i-1}$$

$$n_{1,t+1} = \sum_{i=1}^{\omega} n_{i,t}\, m_i$$

$$\underline{n}_{t+1} = A\, \underline{n}_t$$

$\underline{P_M}$

Leslie matrix

$\underset{\sim}{n}$ ω-element column vector

A $\omega \times \omega$ square matrix

$$A = \begin{pmatrix} m_1 & m_2 & m_3 & \cdots & m_\omega \\ p_1 & 0 & 0 & \cdots & 0 \\ 0 & p_2 & 0 & \cdots & 0 \\ & & & & \\ & 0 & & p_{\omega-1} & 0 \end{pmatrix}$$

Rules of Matrix algebra

$\underline{a} + \underline{b} = \underline{c}$

$c_i = a_i + b_i$

$k\underline{a} = \underline{c}$

$c_i = k a_i$

$\underline{a} \cdot \underline{b} = c$

"dot product"

$c = \sum_{j=1}^{\omega} a_i b_i$

$\underline{a}$ is a row vector

$\underline{b}$ is a column vector

$$\underline{B}\,\underline{a} = \underline{c}$$

B is square matrix

$\underline{a}$ is columnvector

$\underline{c}$ is columnvector

$$c_i = \underline{b}_i \cdot \underline{a}$$

$$= \sum_{j=1}^{w} b_{ij} \cdot a_i$$