

Fresh start:  
Demographic Stochasticity  
at the population level

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particular case  
Binomial survival process  
Poisson birth process  
Discrete time  
"global property" births and  
deaths are independent  
between individuals

# notation

$\alpha$  death probability  
 $\beta$  Poisson birth rate  
(mean and variance)

$N$  population size

$N_\alpha$  number of deaths  
of one time step

$N_\beta$  number of births  
in one time step

$$N_{t+1} = N_t + N_\beta - N_\alpha$$

Mean outcome

$$\bar{N}_\alpha = N\alpha$$

$$\bar{N}_\beta = N\beta$$

Population level process:

$$N_\alpha \sim B(N, \alpha)$$

$$\text{Var}(N_\alpha) = N\alpha(1-\alpha)$$

$$N_\beta \sim P(N\beta)$$
$$\text{Var}(N_\beta) = N\beta$$

define  $\lambda_t = \frac{N_{t+1}}{N_t}$

$$= \frac{N_t + N_\beta - N_\alpha}{N_t}$$
$$= 1 + \frac{N_\beta}{N} - \frac{N_\alpha}{N}$$

$$\bar{\lambda} = 1 + \beta - \alpha$$

$$\begin{aligned}
& \text{Var} \left( 1 + \frac{N_\beta}{N} - \frac{N_\alpha}{N} \right) \\
&= 0 + \frac{\text{Var}(N_\beta)}{N^2} + \frac{\text{Var}(N_\alpha)}{N^2} \\
&= \frac{1}{N^2} \left( N\beta + N(\alpha(1-\alpha)) \right) \\
&= \frac{1}{N} \left( \beta + \alpha(1-\alpha) \right)
\end{aligned}$$

$$\sigma_{\lambda} = \sqrt{\frac{\beta_0 + \alpha(1-\alpha)}{N}}$$

$$\pm 1.96 \sigma_{\lambda} = \pm 1.96 \sqrt{\frac{\beta_0 + \alpha(1-\alpha)}{N}}$$

$$\bar{\lambda} - 1.96 \sigma_{\lambda} = 1 + \beta - \alpha - 1.96 \sqrt{\frac{\beta_0 + \alpha(1-\alpha)}{N}}$$

For 99.5% prob of growth  
in one time step

$$1 + \beta - \alpha - 1.96 \sqrt{\frac{\beta + \alpha(1-\alpha)}{N}} > 1$$

$$\beta - \alpha - 1.96 \sqrt{\frac{\beta + \alpha(1-\alpha)}{N}} > 0$$

for  $\beta = .55$  and  $\alpha = .5$

$$.05 - 1.96 \sqrt{\frac{1.05 - \alpha^2}{N}} > 0$$

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# Conditional probabilities in the PVAX series programs

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Reporting parameters:

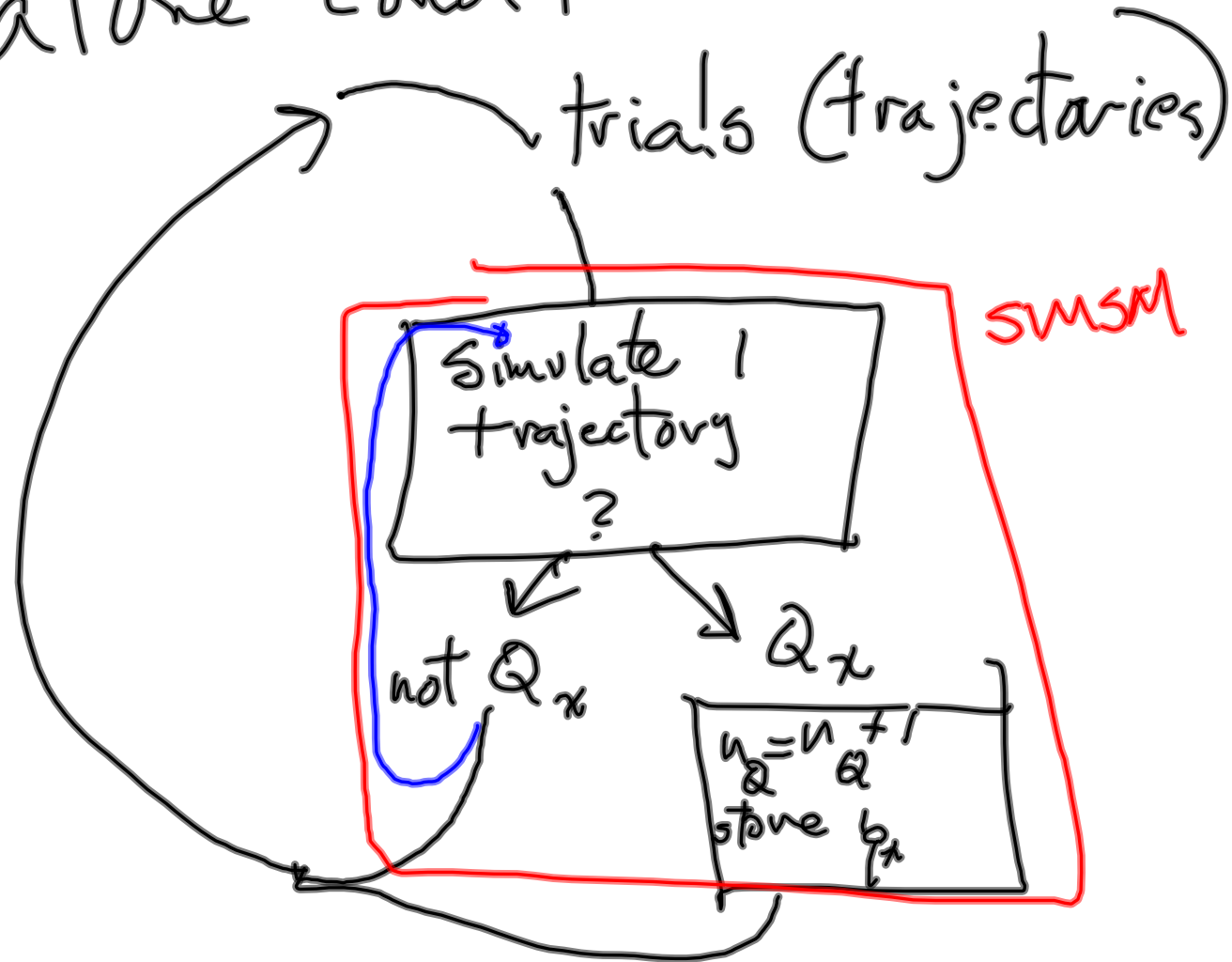
1) probability of quasi-extinction  
by time horizon

code as 1 if trajectory is  
quasi-extinct, code as 0  
if not; read fraction directly  
off marginal summary table



2) time to quasi-extinction  
conditioned on  
quasi-extinction by the  
time horizon

# Flow chart for stand alone conditional



3) Population size at  
time horizon conditioned

on not having gone  
quasi-extinct

(same trick)

# ENVIRONMENTAL VARIATION

\* SAMPLED EACH YEAR

\* AFFECTS ALL  
INDIVIDUALS THE  
SAME WAY

