APPENDICES

APPENDIX A

COMPUTATION OF JACOBIAN OF THE HYPERPARAMETERS

Computation of Jacobian of $\left(\frac{\alpha}{\alpha+\beta},(\alpha+\beta)^{-\frac{1}{2}}\right)$.

$$
\begin{aligned}
J &= \begin{vmatrix} \dfrac{\partial\left(\dfrac{\alpha}{\alpha+\beta}\right)}{\partial\alpha} & \dfrac{\partial\left(\dfrac{\alpha}{\alpha+\beta}\right)}{\partial\beta} \\ \dfrac{\partial(\alpha+\beta)^{-\frac{1}{2}}}{\partial\alpha} & \dfrac{\partial(\alpha+\beta)^{-\frac{1}{2}}}{\partial\beta} \end{vmatrix} \\
&= \begin{vmatrix} \dfrac{\beta}{(\alpha+\beta)^2} & \dfrac{-\alpha}{(\alpha+\beta)^2} \\ -\dfrac{1}{2}(\alpha+\beta)^{-\frac{3}{2}} & -\dfrac{1}{2}(\alpha+\beta)^{-\frac{3}{2}} \end{vmatrix} \\
&= -\frac{\beta}{2}(\alpha+\beta)^{-\frac{7}{2}} - \frac{\alpha}{2}(\alpha+\beta)^{-\frac{7}{2}} \\
&= -\frac{1}{2}(\alpha+\beta)^{-\frac{5}{2}}
\end{aligned}
$$

APPENDIX B

TRANSFORMATION OF THE HYPERPARAMETERS

Transformation of $p(\alpha,\beta)$ to $p\left(\ln\left(\frac{\alpha}{\beta}\right),\ln(\alpha+\beta)\right)$, when $p(\alpha,\beta)\propto(\alpha+\beta)^{-\frac{5}{2}}$.

Let $y_1=\ln\left(\frac{\alpha}{\beta}\right)$ and $y_2=\ln(\alpha+\beta)$. Then the unique solutions for these equations are:

$$\alpha=\frac{e^{y_1}e^{y_2}}{1+e^{y_1}}$$

$$\beta=\frac{e^{y_2}}{1+e^{y_1}}$$

Then the Jacobian is:

$$J=\begin{vmatrix}\frac{\partial\alpha}{\partial y_1} & \frac{\partial\alpha}{\partial y_2}\\ \frac{\partial\beta}{\partial y_1} & \frac{\partial\beta}{\partial y_2}\end{vmatrix}=\begin{vmatrix}\frac{e^{y_1}e^{y_2}}{\left(1+e^{y_1}\right)^2} & \frac{e^{y_1}e^{y_2}}{1+e^{y_1}}\\ \frac{-e^{y_1}e^{y_2}}{\left(1+e^{y_1}\right)^2} & \frac{e^{y_2}}{1+e^{y_1}}\end{vmatrix}$$

$$=\frac{e^{y_1}e^{2y_2}}{\left(1+e^{y_1}\right)^2}=\alpha\beta$$

Therefore, $p\left(\ln\left(\frac{\alpha}{\beta}\right),\ln(\alpha+\beta)\right)=p(\alpha,\beta)|J|\propto\alpha\beta(\alpha+\beta)^{-\frac{5}{2}}$.